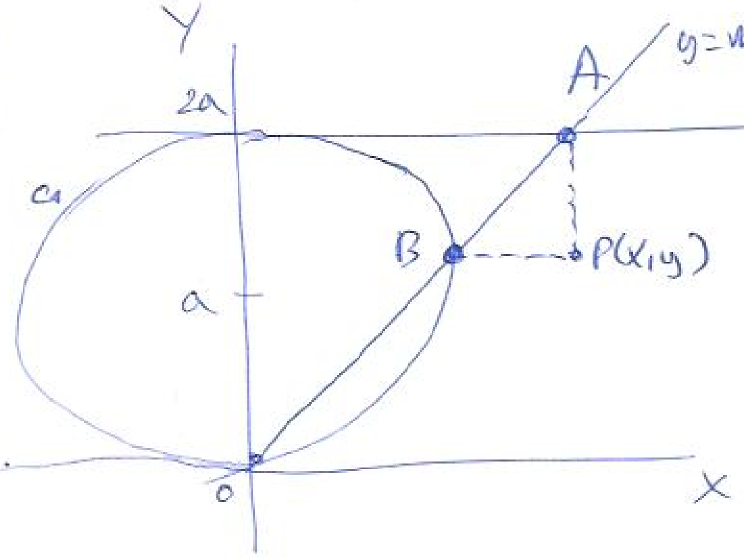
(2º)

a/



$y = mx \Rightarrow m = \frac{y}{x} \Rightarrow m = \frac{2a}{x}$

$y = 2a$

$C_1 : x^2 + (y-a)^2 = a^2$

Coordenadas de A:

$A(x, 2a)$

Coordenadas B:

$x^2 = a^2 - (y-a)^2$

$x = \sqrt{2ay - y^2}$

luego $B(\sqrt{2ay-y^2}, y)$

Tenemos que $\overline{OB}$ es proporcional a $\overline{OA}$ luego:

$$\frac{2a}{x} = \frac{y}{\sqrt{2ay-y^2}} \Rightarrow 2a\sqrt{2ay-y^2} = xy \Rightarrow$$

$$\Rightarrow 4a^2(2ay-y^2) = x^2y^2 \Rightarrow 8a^3\cancel{y} - 4a^2y^{\cancel{2}} = x^2y^{\cancel{2}} \Rightarrow$$

$$\Rightarrow 8a^3 - 4a^2y = x^2y \Rightarrow x^2y + 4a^2y = 8a^3 \Rightarrow$$

$$\Rightarrow \boxed{y = \frac{8a^3}{x^2+4a^2}}$$

b/ Es evidente que f es par o sea $f(x) = f(-x)$ por lo que el área buscada es:

$$A = \int_{-\infty}^{+\infty} \frac{8a^3}{x^2+4a^2}\,dx = 2\int_0^{+\infty} \frac{8a^3}{x^2+4a^2}\,dx = 2\int_0^{+\infty} \frac{\cancel{8}^{2} a^{\cancel{3}}}{\cancel{4a^2}\left[1+\left(\frac{x}{2a}\right)^2\right]}\,dx$$

$$= 4a\int_0^{+\infty} \frac{1/2a}{1/2a\left[1+\left(\frac{x}{2a}\right)^2\right]}\,dx = 8a^2\int_0^{+\infty} \frac{1/2a}{1+\left(\frac{x}{2a}\right)^2}\,dx =$$

$$= 8a^2 \cdot \left[\operatorname{arctg}\frac{x}{2a}\right]_0^{+\infty} = 8a^2\left(\frac{\pi}{2} - 0\right) = \underline{4a^2\pi \text{ u.}}$$